# Principles Of Chemical Engineering Calculations 

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## Lecture 13

## Product separation and Recycle

Recycle is fed back from a downstream unit to an upstream unit, as shown in Figure. The stream containing the recycled material is known as a recycle stream.

Recycle system is a system that includes one or more recycle streams.


## Recycle without Chemical Reaction

## Example 13

Figure below is a schematic of a process for the production of flake NaOH . The fresh feed to the process is $10,000 \mathrm{lb} / \mathrm{hr}$ of a $40 \%$ aqueous NaOH solution.

a. Determine the flow rate of water removed by the evaporator(W), and the recycle rate for this process (R).
b. Assume that the same production rate of NaOH flakes occurs, but the filtrate is not recycled. What would be the total feed rate of $40 \% \mathrm{NaOH}$ have to be then? Assume that the product solution from the evaporator still contains $50 \% \mathrm{NaOH}$.

## Solution

Open, steady-state process.
a. Basis: $10,000 \mathrm{lb}$ fresh feed (equivalent to 1 hour)

The unknowns are W, G, P, and R.

## Overall NaOH mass balance

input $=$ output
$(0.4)(10,000)=0.95 \mathrm{P}+(0.45)(0.05) \mathrm{P}$
$\mathrm{P}=4113 \mathrm{lb}$

## Overall $\mathrm{H}_{2} \underline{\mathrm{O} \text { mass balance }}$

input $=$ output
$(0.6)(10,000)=\mathrm{W}+[(0.55)(0.05)](4113)$
$\mathrm{W}=5887 \mathrm{lb}$
The total amount of NaOH exiting with P is $[(0.95)+(0.45)(0.05)](4113)$
$=4000 \mathrm{lb}$

## $\underline{\mathrm{NaOH} \text { mass balance on the crystallizer }}$

$0.5 \mathrm{G}=4000+0.45 \mathrm{R}$

## $\underline{\mathrm{H}}_{2} \underline{\mathrm{O}}$ mass balance on the crystallizer

$0.5 \mathrm{G}=113+0.55 \mathrm{R}$

Solving equations 1 and 2 gives:
$\mathrm{R}=38,870 \mathrm{lb}$
b. The basis is now $\mathbf{P}=4113 \mathrm{lb}$ (the same as $\mathbf{1}$ hour)


The unknowns are now F, W, G, and H.

## NaOH mass balance on the crystallizer

$0.5 \mathrm{G}=[(0.95)+(0.05)(0.45)](4113)+0.45 \mathrm{H}$

## $\underline{H}_{2} \underline{O}^{\mathbf{O} \text { mass balance on the crystallizer }}$

$0.5 \mathrm{G}=[(0.05)(0.55)(4113)]+0.55 \mathrm{H}$
By solving equations 1 and 2 gives:
$\mathrm{H}=38,870 \mathrm{lb}$

## Overall NaOH mass balance

$0.40 \mathrm{~F}=0.45(38,870)+4000$
$\mathrm{F}=53,730 \mathrm{lb}$

Note that without recycle, the feed rate must be 5.37 times larger than with recycle to produce the same amount of product.

## Recycle with Chemical Reaction

The most common application of recycle for systems involving chemical reaction is the recycle of reactants, an application that is used to increase the overall conversion in a reactor. Figure below shows a simple example for the reaction $\quad A \rightarrow B$


If you calculate the extent of reaction for the overall process based on B
$\xi_{\text {overall }}=\frac{100-0}{1}=100$ moles reacting
If you use material balances to calculate the output P of the reactor(on the basis of 1 second) you get $A=900 \mathrm{~g}$ mol and $\mathrm{B}=100 \mathrm{~g}$ mol

And the extent of reaction based on B for the reactor by itself as the system is:
$\xi_{\text {reactor }}=\frac{100-0}{1}=100$ moles reacting
Two types of conversion when reactions occur:

1. Overall fraction conversion: reactant input to process -reactant output from process
reactant input to process
2. Single - pass fraction conversion:
$\underline{\text { reactant input to reactor -reactant output from reactor }}$
reactant input to reactor

For the simple recycle reactor in Figure , the overall conversion is:

$$
\frac{100-0}{100} * 100=100 \%
$$

And the single-pass conversion is:
$\frac{1000-900}{1000} * 100=10 \%$
The overall conversion and the single-pass conversion can be expressed in terms of the extent of reaction, $\xi$.
Overall conversion of species $\mathrm{A}=f_{\mathrm{OA}}=\frac{-v_{\mathrm{A}} \xi}{\mathrm{n}_{\mathrm{A}}^{\text {process feed }}}$
Single pass conversion of species $\mathrm{A}=f_{\mathrm{SP}}=\frac{-v_{\mathrm{A}} \xi}{\mathrm{n}_{\mathrm{A}}^{\text {reactor feed }}}$

## Example 14

Cyclohexane $\left(\mathrm{C}_{6} \mathrm{H}_{12}\right)$ can be made by the reaction of benzene $(\mathrm{Bz})$ $\left(\mathrm{C}_{6} \mathrm{H}_{6}\right)$ with hydrogen according to the following reaction:
$\mathrm{C}_{6} \mathrm{H}_{6}+3 \mathrm{H}_{2} \rightarrow \mathrm{C}_{6} \mathrm{H}_{12}$

For the process shown in Figure below, determine the ratio of the recycle stream to the fresh feed stream if the overall conversion of benzene is $95 \%$, and the single-pass conversion is $20 \%$. Assume that $20 \%$ excess hydrogen is used in the fresh feed, and that the composition of the recycle stream is 22.74 mol \% benzene and 77.26 mol \% hydrogen.


## Solution

The process is open and steady state.
Basis $=100$ mol of fresh benzene feed
excess $\mathrm{H}_{2}=\frac{\text { in-required }}{\text { required }}$
In $\mathrm{H}_{2}$ (Feed):
$0.2=\frac{\mathrm{in}-3(100)}{3(100)}$
In $\mathrm{H}_{2}=360 \mathrm{~mol}$
The total process feed $=100+360=460 \mathrm{~mol}$
From Equation of overall conversion for benzene ( $v_{\mathrm{BZ}}=-1$ ):
$0.95=\frac{-(-1) \xi}{100}$
$\xi=95$ reacting moles.
The unknowns are R, $n_{B z}^{P}, n_{H_{2}}^{P}$ and $n_{C_{6} H_{12}}^{P}$
The species overall balances are:
$n_{i}^{\text {out }}=n_{i}^{\text {in }}+v_{i} \xi_{\text {overall }}$
Bz overall balance
$n_{B Z}^{P}=100+(-1)(95)=5 \mathrm{~mol}$
$\underline{H}_{\underline{2}}$ overall balance
$n_{H_{2}}^{P}=360+(-3)(95)=75 \mathrm{~mol}$
$\underline{\mathrm{C}}_{6} \underline{\mathrm{H}}_{12}$ overall balance
$n_{C_{6} H_{12}}^{P}=0+(1)(95)=95 \mathrm{~mol}$
$\mathrm{P}=5+75+95=175 \mathrm{~mol}$

The amount of the Bz feed to the reactor is $100+0.2274 \mathrm{R}$, and $\xi=95$. Thus, for benzene:
$0.2=\frac{-(-1) 95}{100+0.227 \mathrm{R}}$
$\mathrm{R}=1649 \mathrm{~mol}$

Finally, the ratio of recycle to fresh feed is:
$\frac{\mathrm{R}}{\mathrm{F}}=\frac{1649 \mathrm{~mol}}{460 \mathrm{~mol}}=3.58$

## Lecture 14

## Bypass and Purge

1. A bypass stream-a stream that skips one or more stages of the process an goes directly to another downstream stage .


A bypass stream can be used to control the composition of a final exit stream from a unit by mixing the bypass stream and the unit exit stream in suitable proportions to obtain the desired final composition.
2. A purge stream - a stream bled off from the process to remove an accumulation of inert or unwanted material that might otherwise build up in the recycle stream.


## Example 15

In the feedstock preparation section of a plant manufacturing natural gasoline, isopentane is removed from butane-free gasoline. What fraction of the butane-free gasoline is passed through the isopentane tower? The process is in the steady state and no reaction occurs.


## Solution

Basis: 100 kg feed
Overall balances

1. Total material balance:

$$
\begin{aligned}
& \text { In }=\text { out } \\
& 100=S+P
\end{aligned}
$$

2. Component balance for $n-\mathrm{C}_{5}$ (tie component)

$$
\begin{aligned}
& \text { In = out } \\
& 100(0.8)=S(0)+P(0.9)
\end{aligned}
$$

Consequently,
$P=100\left(\frac{0.8}{0.9}\right)=88.9 \mathrm{~kg}$
$\mathrm{S}=100-88.9=11.1 \mathrm{~kg}$

## Balance around isopentane tower:

Let x be the kg of butane-free gas going to the isopentane tower, and y be the kg of the $\mathrm{n}-\mathrm{C}_{5} \mathrm{H}_{12}$ stream leaving the isopentane tower.
3. Total material balance:

In = Out
$\mathrm{x}=11.1+\mathrm{y}$
4. Component balance for $\mathrm{n}-\mathrm{C}_{5}$, $x(0.80)=y$

Consequently, combining (3) and (4) yields :
$\mathrm{x}=55.5 \mathrm{~kg}$, or the desired fraction is 0.55 .

## Problems

1. Sea water is to be desalinized by reverse osmosis using the scheme indicated in Figure. Use the data given in the figure to determine:
(a) the rate of waste brine removal (B)
(b) the rate of desalinized water (called potable water) production(P)
(c) the fraction of the brine leaving the reverse osmosis cell (which acts in essence as a separator) that is recycled.

2. A catalytic dehydrogenation process, produces 1,3 butadiene $\left(\mathrm{C}_{4} \mathrm{H}_{6}\right)$ from pure normal butane $\left(\mathrm{C}_{4} \mathrm{H}_{10}\right)$. The product stream contains $75 \mathrm{~mol} / \mathrm{h}$ of $\mathrm{H}_{2}$ and $13 \mathrm{~mol} / \mathrm{h}$ of $\mathrm{C}_{4} \mathrm{H}_{10}$ as well as $\mathrm{C}_{4} \mathrm{H}_{6}$. The recycle stream is $30 \%(\mathrm{~mol}) \mathrm{C}_{4} \mathrm{H}_{10}$ and $70 \%(\mathrm{~mol}) \mathrm{C}_{4} \mathrm{H}_{6}$ and the flow is $24 \mathrm{~mol} / \mathrm{h}$.
a. What is the feed rate, $F$, and the product flow rate of $\mathrm{C}_{4} \mathrm{H}_{6}$ leaving the process?
b. What is the single-pass conversion of butane in the process?

3. To save energy, stack gas from a furnace is used to dry rice. What is the amount of recycle gas (in lb mol) per 100 lb of P if the concentration of water in the gas stream entering the dryer is $5.2 \%$ ?


## Lecture 15

## Gases, Vapors, Liquids

- Property is meaning any measurable characteristic of a substance, such as pressure, volume, or temperature, or a characteristic that can be calculated or deduced, such as internal energy.
- State : A system will possess a unique set of properties, such as temperature, pressure, density, and so on, at a given time, and thus is said to be in a particular state. A change in the state of the system results in a change in at least one of its properties.
- By equilibrium : we mean a state in which there is no tendency toward spontaneous change. When a system is in equilibrium with another system, or the surroundings, it will not change its state unless the other system, or the surroundings, also changes.
- A phase is defined as a completely homogeneous and uniform state of matter. Liquid water would be a phase; ice would be another phase. Two immiscible liquids in the same container, such as mercury and water, would represent two different phases because the liquids have different properties.


### 3.1 Ideal Gas Law Calculations

## The Ideal Gas Law

Under conditions such that the average distance between the molecules in a substance is great enough to neglect the effect of the intermolecular forces and the volume of the molecules themselves, a gas can be termed an ideal gas.

More properly, an ideal gas is an imaginary gas that obeys exactly the following relationship:
$p V=n R T$
Where $\mathrm{p}=$ absolute pressure of the gas
$\mathrm{V}=$ total volume occupied by the gas
$\mathrm{n}=$ number of moles of the gas
$\mathrm{R}=$ ideal gas constant in appropriate units
$\mathrm{T}=$ absolute temperature of the gas
Sometimes the ideal gas law is written as
$p \widehat{V}=R T$
Where $\hat{V}$ is the specific volume (volume per mole or mass) of the gas.

| Table 1 Common Standard Conditions for the Ideal Gas |  |  |  |
| :---: | :---: | :---: | :---: |
| System | $T$ | $P$ | $\hat{V}$ |
| SI | 273.15 K | 101.325 kPa | $22.415 \mathrm{~m}^{3} / \mathrm{kg} \mathrm{mol}$ |
| Universal scientific | $0.0^{\circ} \mathrm{C}$ | 760 mm Hg | 22.415 liters $/ \mathrm{g} \mathrm{mol}$ |
| Natural gas industry | $60.0^{\circ} \mathrm{F}$ | 14.696 psia | $379.4 \mathrm{ft}^{3} / \mathrm{lb} \mathrm{mol}$ |
|  | $\left(15.0^{\circ} \mathrm{C}\right)$ | $(101.325 \mathrm{kPa})$ |  |
| American engineering | $32^{\circ} \mathrm{F}$ | 1 atm | $359.05 \mathrm{ft}^{3} / \mathrm{lb} \mathrm{mol}$ |

## Example 1

Calculate the volume, in cubic meters, occupied by 40 kg of $\mathrm{CO}_{2}$ at standard conditions.

## Solution

Basis: 40 kg of $\mathrm{CO}_{2}$
$40 \mathrm{~kg} \mathrm{CO}_{2}\left|\frac{1 \mathrm{kmol} \mathrm{CO}}{44 \mathrm{~kg} \mathrm{CO}} \mathrm{K}_{2}\right| \frac{22.42 \mathrm{~m}^{3} \mathrm{CO}_{2}}{1 \mathrm{kmol} \mathrm{CO}}=20.4 \mathrm{~m}^{3} \mathrm{CO}_{2}$ at S.C.

## Example 2

Find the value for the universal gas constant R for the following combinations of units: For 1 mol of ideal gas when the pressure is in atm, the volume in $\mathrm{cm}^{3}$, and the temperature in K .

## Solution

at standard conditions we will use the approximate values:
$\mathrm{p}=1 \mathrm{~atm}$
$\hat{V}=22415 \mathrm{~cm}^{3} / \mathrm{mol}$
$T=273.15 \mathrm{~K}$
$R=\frac{p \widehat{V}}{T}=\frac{1 \mathrm{~atm}}{273.15 \mathrm{~K}} \left\lvert\, \frac{22415 \mathrm{~cm}^{3}}{1 \mathrm{~mol}}=82.06 \frac{\left(\mathrm{~cm}^{3}\right)(\mathrm{atm})}{(K)(\mathrm{mol})}\right.$
In many processes going from an initial state to a final state, you can use the ratio of ideal gas law in the respective states and eliminate $R$ as follows (the subscript 1 designates the initial state, and the subscript 2 designates the final state);
$\frac{p_{1} V_{1}}{p_{2} V_{2}}=\frac{n_{1} R T_{1}}{n_{2} R T_{2}}$
$\left(\frac{p_{1}}{p_{2}}\right)\left(\frac{V_{1}}{V_{2}}\right)=\left(\frac{n_{1}}{n_{2}}\right)\left(\frac{T_{1}}{T_{2}}\right)$

## Example 3

Calculate the volume occupied by 88 lb of $\mathrm{CO}_{2}$ at a pressure of 32.2 ft of water and at $15^{\circ} \mathrm{C}$.

## Solution


$\left(\frac{p_{1}}{p_{2}}\right)\left(\frac{V_{1}}{V_{2}}\right)=\left(\frac{n_{1}}{n_{2}}\right)\left(\frac{T_{1}}{T_{2}}\right)$
$V_{2}=V_{1}\left(\frac{p_{1}}{p_{2}}\right)\left(\frac{T_{2}}{T_{1}}\right)$
Assume that the pressure is absolute pressure.

|  | At state 1 (S.C) | At state 2 |
| :--- | :--- | :--- |
| Pressure $\left(\mathrm{ft} \mathrm{H}_{2} \mathrm{O}\right)$ | 33.91 | 32.2 |
| Temperature $(\mathrm{K})$ | 273 | $15+273=288$ |

Basis: 88 lb of $\mathrm{CO}_{2}$

$$
V_{2}=88 l b \mathrm{CO}_{2}\left|\frac{1 \mathrm{lb} \mathrm{~mol} \mathrm{CO}}{2}\right| \frac{359 \mathrm{ft}^{3}}{44 \mathrm{lb} \mathrm{CO}_{2}}\left|\frac{288}{1 \mathrm{lb} \mathrm{~mol}}\right| \frac{33.91}{273}=798 \mathrm{ft}^{3} \mathrm{CO}_{2}
$$



## The Gas Density

The density of a gas is defined as the mass per unit volume and can be expressed in kilograms per cubic meter, pounds per cubic foot, grams per liter, or other units

## Example 4

What is the density of $\mathrm{N}_{2}$ at $27^{\circ} \mathrm{C}$ and 100 kPa in SI units?

## Solution

Basis: $1 \mathrm{~m}^{3}$ of $\mathrm{N}_{2}$ at $27^{\circ} \mathrm{C}$ and 100 kPa

$$
1 \mathrm{~m}^{3}\left|\frac{273 \mathrm{~K}}{300 \mathrm{~K}}\right| \frac{100 \mathrm{kPa}}{101.3 \mathrm{kPa}}\left|\frac{1 \mathrm{kmol}}{22.4 \mathrm{~m}^{3}}\right| \frac{28 \mathrm{~kg}}{1 \mathrm{kmol}}=1.123 \mathrm{~kg}
$$

Density $=1.123 \mathrm{~kg} / \mathrm{m}^{3}$ of $\mathrm{N}_{2}$ at $27^{\circ} \mathrm{C}$ and 100 kPa

## The Gas Specific Gravity

The specific gravity of a gas is usually defined as the ratio of the density of the gas at a desired temperature and pressure to that of air (or any specified reference gas) at a certain temperature and pressure.

## Example 5

What is the specific gravity of $\mathrm{N}_{2}$ at $80^{\circ} \mathrm{F}$ and 745 mm Hg compared to air at $80^{\circ} \mathrm{F}$ and 745 mm Hg ?

## Solution

Basis: $\mathrm{Ift}{ }^{3}$ of air at $80^{\circ} \mathrm{F}$ and 745 mm Hg
$1\left|\frac{492}{540}\right| \frac{745}{760}\left|\frac{1}{359}\right| \frac{29}{1}=0.0721 \frac{l b}{f t^{3}}$ at $80^{\circ} \mathrm{F}$ and 745 mm Hg
Basis: $1 \mathrm{ft}^{3}$ of $\mathrm{N}_{2}$ at $80^{\circ} \mathrm{F}$ and 745 mm Hg
$1\left|\frac{492}{540}\right| \frac{745}{760}\left|\frac{1}{359}\right| \frac{28}{1}=0.0697 \frac{l b}{f t^{3}}$ at 80 의 and 745 mm Hg
$(s p . g r .)_{N_{2}}=\frac{0.0697}{0.0721}=0.0967 \frac{\mathrm{lb} N_{2} / f t^{3}}{l b \text { air } / f t^{3}}$

## Ideal Gas Mixtures and Partial Pressure

The partial pressure of gas; i defined by Dalton, $\mathrm{p}_{\mathrm{i}}$, namely the pressure that would be exerted by a single component in a gaseous mixture if it existed by itself in the same volume as occupied by the mixture and at the same temperature of the mixture is:
$p_{i} V_{\text {total }}=n_{i} R T_{\text {total }}$
Where $p_{i}$ is the partial pressure of component i . If you divide Eq. (3) by Eq. (1), you find that:
$\frac{p_{i} V_{\text {total }}}{p_{\text {total }} V_{\text {total }}}=\frac{n_{i} R T_{\text {total }}}{n_{\text {total }} R T_{\text {total }}}$
Or, $p_{i}=p_{\text {total }} \frac{n_{i}}{n_{\text {totl }}}=p_{\text {total }} y_{i}$
Where $y_{i}$ is the mole fraction of component $i$.
Can you show that Dalton's law of the summation of partial pressures is true using Eq.4?
$p_{1}+p_{2}+\cdots+p_{n}=p_{t}$

## Example 6

A flue gas analyzes $14.0 \% \mathrm{CO}_{2}, 6.0 \% \mathrm{O}_{2}$, and $80.0 \% \mathrm{~N}_{2}$. It is at $400^{\circ} \mathrm{F}$ and 765.0 mm Hg pressure. Calculate the partial pressure of each component.

## Solution

Use Eq, (4) $\mathrm{p}_{\mathrm{i}}=\mathrm{p}_{\mathrm{t}} \mathrm{y}_{\mathrm{i}}$
Basis: 1.00 kg (or lb) mol flue gas

| Component | $k g($ or lb) mol | $p(\mathrm{~mm} \mathrm{Hg}$ ) |
| :---: | :---: | :---: |
| $\mathrm{CO}_{2}$ | 0.140 | 107.1 |
| $\mathrm{O}_{2}$ | 0.060 | 45.9 |
| $\mathrm{~N}_{2}$ | 0.800 | 612.0 |
| Total | 1.000 | 765.0 |

## Lecture 16

## Real Gas Relationships

We have said that at room temperature and pressure many gases can be assumed to act as ideal gases.

However, for some gases under normal conditions, and for most gases under conditions of high pressure, values of the gas properties that you might obtain using the ideal gas law would be at wide variance with the experimental evidence

Thus it is clear that we need some way of computing the $\mathrm{p}-\mathrm{V}-\mathrm{T}$ properties of a gas that is not ideal, i.e. the real gas.

Three methods of getting or predicting real gas properties in lieu of having experimental data:

1. Compressibility charts
2. Equations of state
3. Estimated properties

Critical State, Reduced Parameters, and Compressibility
The critical state for the gas-liquid transition is the set of physical conditions at which the density and other properties of the liquid and vapor become identical.

A supercritical fluid, that is, a compound in a state above the critical point, combines some of the properties of both gases and liquids.

Another set of terms with which you should immediately become familiar are the reduced parameters. These are corrected, or normalized, conditions of temperature, pressure, and volume and are expressed mathematically as:
$T_{r}=\frac{T}{T_{c}}$,
$P_{r}=\frac{P}{P_{c}}$,
$V_{r}=\frac{V}{V_{c}}$

## Compressibility Factor (z)

One common way is to modify the ideal gas law by inserting an adjustable coefficient $z$, the compressibility factor, a factor that compensates for the non ideality of the gas. Thus, the ideal gas law becomes a real gas law, a generalized equation of state:
$p V=z n R T$

Figure below show the generalized compressibility charts or z - factor chart prepared by Nelson and Obert.

One of these helpful parameters is the ideal reduced volume defined as:
$V_{r i}=\frac{\widehat{V}}{\widehat{V}_{c i}}$
$V_{c i}$ the ideal critical volume. Or,
$\widehat{V}_{c i}=\frac{R T_{c}}{p_{c}}$


Generalized Compressibility Chart, Medium Pressure.

## Note

The value $\mathrm{z}=1$ represents ideality, and the value $\mathrm{z}=0.27$ is the compressibility factor at the critical point.

## Example 7

for the system below, a tank of volume $120 \mathrm{ft}^{3}$ contains 125 lb of $\mathrm{NH}_{3}$ gas at 292 psig and $125^{\circ} \mathrm{F}$. the specific volume calculated was $1.20 \mathrm{ft}^{3} / \mathrm{lb}$ and hence there are only 100 lb of $\mathrm{NH}_{3}$ in the tank. Is this correct?

## Solution

Based on ideal gas law for I lb:
$R=10.73 \frac{(\mathrm{psia})\left(\mathrm{ft}^{3}\right)}{(\mathrm{lb} \mathrm{mol})\left({ }^{( } \mathrm{R}\right)}$
$\mathrm{p}=292+14.7=306.7 \mathrm{psia}$
$\mathrm{T}=125^{\circ} \mathrm{F}+460=585^{\circ} \mathrm{R}$
$n=\frac{1 \mathrm{lb}}{17 \mathrm{lb} / \mathrm{lb} \mathrm{mol}}=\frac{1}{17} \mathrm{lb} \mathrm{mol}$
$\widehat{V}=\frac{R T}{p}=\frac{\frac{1}{17}(10.73)(585)}{306.7}=1.20 \mathrm{ft}^{3} / \mathrm{lb}$
However, he should have used the compressibility factor, because $\mathrm{NH}_{3}$ does not behave as an ideal gas under the observed conditions of temperature and pressure. Let us again compute the mass of gas in the tank this time using:
$p V=z n R T$
From Appendix D: $T_{c}=729.9 R, p_{c}=1636$ psia

$$
T_{r}=\frac{585 R}{729.9 R}=0.801 \quad, P_{r}=\frac{306.7 \text { psia }}{1636 \text { psia }}=0.187
$$

From figure 3 , you can read $\mathrm{z}=0.855$. Now $\hat{V}$ can be calculated as:

$$
\widehat{V}=\frac{1.20 \mathrm{ft}^{3} \text { ideal }}{l b} \left\lvert\, \frac{0.855}{1}=1.03 \mathrm{ft}^{3} / \mathrm{lb}_{3} N H_{3}\right.
$$

$\left.\frac{1 \mathrm{lbNH} H_{3}}{1.03 \mathrm{ft}^{3}} \right\rvert\, 120 \mathrm{ft}^{3}=117 \mathrm{lb} \mathrm{NH}_{3}$

## 2. Equations of State

Equations of state relate the p-V-T properties of a pure substance (or mixtures) by theoretical or empirical relations.

The simplest example of an equation of state is the ideal gas law itself.

## Van der Waals equation

$p=\frac{n R T}{V-n b}-\frac{n^{2} a}{V^{2}}$
$a=\left(\frac{27}{64}\right) \frac{R^{2} T_{c}^{2}}{P_{c}}$
$b=\left(\frac{1}{8}\right) \frac{R T_{c}}{P_{c}}$

## Example 8

A cylinder $0.150 \mathrm{~m}^{3}$ in volume containing 22.7 kg of propane $\mathrm{C}_{3} \mathrm{H}_{8}$ stands in the hot sun. A pressure gauge shows that the pressure is 4790 kPa gauge. What is the temperature of the propane in the cylinder? Use van der Waals equation.

## Solution

Basis: 22.7 kg of propane
From handbook, $\quad a=9.24 * 10^{6} \mathrm{~atm}\left(\frac{\mathrm{~cm}^{3}}{\mathrm{~mol}}\right)^{2}, \quad b=90.7 \frac{\mathrm{~cm}^{3}}{\mathrm{~mol}}$
$\left(p+\frac{n^{2} a}{V^{2}}\right)(V-n b)=n R T$
$\mathrm{p}=(4790+101) \mathrm{kPa} \left\lvert\, \frac{1 \mathrm{~atm}}{101.3 \mathrm{kPa}}=48.3 \mathrm{~atm}\right.$ abs.
$R=82.06 \frac{\left(\mathrm{~cm}^{3}\right)(\mathrm{atm})}{(\mathrm{mol})(\mathrm{K})}$
$\mathrm{n}=\frac{22.7 \mathrm{~kg}}{44 \mathrm{~kg} / \mathrm{kmol}}=0.516 \mathrm{kmol}$ propane
$\left[48.3+\frac{\left(0.516 * 10^{3}\right)^{2}\left(9.24 * 10^{6}\right)}{\left(0.150 * 10^{6}\right)^{2}}\right]\left(0.150 * 10^{6}-\left(0.516 * 10^{3}\right) *(90.7)\right)=$ $\left(0.516 * 10^{3}\right)(82.06)\left(T_{K}\right)$
$T=384 K$
Gaseous mixture
$p_{c}^{\prime}=p_{c_{A}} y_{A}+p_{c_{B}} y_{B}+\cdots$
$T_{c}^{\prime}=T_{c_{A}} y_{A}+T_{c_{B}} y_{B}+\cdots$
$p_{r}^{\prime}=\frac{p}{p_{c}^{\prime}}$
$T_{r}^{\prime}=\frac{T}{T_{c}^{\prime}}$

## Example 9

A gaseous mixture has the following composition (in mole percent):
Methane, $\mathrm{CH}_{4}$ 20

Ethylene, $\mathrm{C}_{2} \mathrm{H}_{4}$
Nitrogen, $\mathrm{N}_{2}$ 50
at 90 atm pressure and $100^{\circ} \mathrm{C}$. Compare the volume per mole as computed by the methods of:
(a) the perfect gas law
(b) the pseudoreduced technique (Kay's method)

## Solution

Basis: 1 mol of gas mixture

| Component | $\mathbf{T}_{\mathbf{c}}(\mathbf{K})$ | $\mathbf{p}_{\mathbf{c}}(\mathbf{a t m})$ |
| :--- | :--- | :--- |
| $\mathrm{CH}_{4}$ | 191 | 45.8 |
| $\mathrm{C}_{2} \mathrm{H}_{4}$ | 283 | 50.5 |
| $\mathrm{~N}_{2}$ | 126 | 33.5 |

$$
R=82.06 \frac{\left(\mathrm{~cm}^{3}\right)(\mathrm{atm})}{(\mathrm{mol})(\mathrm{K})}
$$

(a) Perfect gas law:

$$
\widehat{V}=\frac{R T}{p}=\frac{1(82.06)(373)}{90}=340 \mathrm{~cm}^{3}
$$

(b)

$$
\begin{aligned}
& p_{c}^{\prime}=p_{c_{A}} y_{A}+p_{c_{B}} y_{B}+p_{c_{c}} y_{C}=(45.8)(0.2)+(50.5)(0.3)+ \\
& (33.5)(0.5)=41.2 \mathrm{~atm} \\
& T_{c}^{\prime}=T_{c_{A}} y_{A}+T y_{B}+T_{c_{C}} y_{C}=(191)(0.2)+(283)(0.3)+ \\
& (126)(0.5)=186 \mathrm{~K} \\
& T_{r}^{\prime}=\frac{T}{T_{c}^{\prime}}=\frac{373}{186}=2.01 \\
p_{r}^{\prime}= & \frac{p}{p_{c}^{\prime}}=\frac{90}{41.2}=2.18
\end{aligned}
$$

From figure 3, $\mathrm{z}=0.965$
$\widehat{V}=\frac{z R T}{p}=\frac{0.965(1)(82.06)(373)}{90}=328 \mathrm{~cm}^{3}$

## Lecture 17

## Vapor Pressure and Liquids

- The word vapor will be reserved to describe a gas below its critical point in a process
- The word gas or non-condensable gas will be used to describe a gas above the critical point or a gas in a process in which it cannot condense.
- If the vapor and liquid of a pure component are in equilibrium, then the equilibrium pressure is called the vapor pressure .
- Any substance has an infinite number of boiling points, but by custom we say the "normal" boiling point is the temperature at which boiling takes place under a pressure of 1 atm (101.3 kPa, 760 mm Hg ).
- When the solid passes directly into the vapor phase without first melting to become a liquid it is said to sublime.


## Change of Vapor Pressure with Temperature

We will use the Antoine equation- it has sufficient accuracy for our needs:
$\ln p^{*}=A-\frac{B}{C+T}$
Where A, B , C are constants different for each substance and T is a temperature, K

## Example 10

Calculate the vapor pressure of benzene at $50^{\circ} \mathrm{C}$ using the Antoine Equation. Also estimate the normal boiling point of benzene (the vapor pressure at 1 atm).

## Solution

$\ln p^{*}=A-\frac{B}{C+T}$
From Appendix G, the coefficients are:
$A=15.9008$,
$B=2788.51$,
C = -52. 36
$\mathrm{p}^{*}$ in mm Hg and T in K .
a. Vapor pressure of benzene at $50^{\circ} \mathrm{C}$ is:

$$
\begin{aligned}
& \ln p^{*}=15.9008-\frac{2788.51}{-52.36+(50+273)} \\
& \mathrm{p}^{*}=270 \mathrm{mmHg} \text { abs. }
\end{aligned}
$$

b. At the boiling point, the vapor pressure is 1 atm ( 760 mmHg abs.)

$$
\begin{aligned}
& \ln (760)=15.9008-\frac{2788.51}{-52.36+T} \\
& \mathrm{~T}=353.3 \mathrm{~K}
\end{aligned}
$$

## Saturation

The volume of the air plus the water vapor increases until the air is saturated with water vapor, after which stage the volume remains constant.

Assuming that the ideal gas law applies to both air and water vapor with excellent precision, we can say that the following relations hold at saturation:

$$
\begin{aligned}
& \frac{\mathrm{p}_{\text {air }} \mathrm{V}}{\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}} \mathrm{~V}}=\frac{\mathrm{n}_{\text {air }} \mathrm{RT}}{\mathrm{n}_{\mathrm{H}_{2} \mathrm{O}} \mathrm{RT}} \\
& \frac{\mathrm{p}_{\text {air }}}{\mathrm{p}_{\mathrm{H}_{2} \mathrm{O}}}=\frac{\mathrm{n}_{\text {air }}}{\mathrm{n}_{\mathrm{H}_{2} \mathrm{O}}}=\frac{\mathrm{p}_{\text {air }}}{\mathrm{p}_{\text {total }}-\mathrm{p}_{\text {air }}}
\end{aligned}
$$

## Example 11

What is the minimum number of cubic meters of dry air at $20^{\circ} \mathrm{C}$ and 100 kpa necessary to evaporate 6.0 kg of ethyl alcohol if the total pressure remains constant at 100 kpa and the temperature remains $20^{\circ} \mathrm{C}$ ? Assume that the air is blown through the alcohol to evaporate it in such away that the exit pressure of the air alcohol mixture is at 100 kPa . (Given: $\mathrm{p}^{*}$ alcohol at $20^{\circ} \mathrm{C}=5.93 \mathrm{kPa}$, mol. wt. ethyl alcohol $=46.07$ ).

## Solution

Assume that the process is isothermal.


Basis: 6 kg of alcohol
$\frac{\mathrm{p}_{\text {alcohol }}}{\mathrm{p}_{\text {air }}}=\frac{\mathrm{n}_{\text {alcohol }}}{\mathrm{n}_{\text {air }}}$
$p_{\text {air }}=p_{\text {total }}-p_{\text {alco hol }}^{*}=(100-5.93) \mathrm{kPa}=94.07 \mathrm{kPs}$
6 kg alcohol $\left|\frac{1 \mathrm{kmol} \text { alco hol }}{46.07 \mathrm{~kg} \text { alco hol }}\right| \frac{94.07 \mathrm{kmol} \text { air }}{5.93 \mathrm{kmol} \text { alco hol }}=2.07 \mathrm{kmol}$ air

$$
V_{\text {air }}=2.07 \mathrm{kmol} \text { air }\left|\frac{8.314(\mathrm{kPa})\left(\mathrm{m}^{3}\right)}{(\mathrm{kmol})(\mathrm{K})}\right| \frac{293 \mathrm{~K}}{100 \mathrm{kPa}}=50.3 \mathrm{~m}^{3}
$$

at $20^{\circ} \mathrm{C}$ and 100 kPa

## Vapor-Liquid Equilibria for Multicomponent Systems

In two-phase vapor-liquid mixture at equilibrium, a component in one phase is in equilibrium with the same component in the other phase. The equilibrium relationship depends on the temperature, pressure, and composition of the mixture.

(a) Constont Pressure

Henry's law: Used primarily for a component whose mole fraction approaches zero, such as a dilute gas dissolved in a liquid:
$\mathrm{p}_{\mathrm{i}}=\mathrm{H}_{\mathrm{i}} \mathrm{X}_{\mathrm{i}}$
Where $\mathrm{p}_{\mathrm{i}}$ is the pressure in the gas phase of the dilute component at equilibrium at some temperature, $\mathrm{x}_{\mathrm{i}}$ is mol fraction of component i in the liquid phase and $\mathrm{H}_{\mathrm{i}}$ is the Henry's law constant.
$y_{i}=\frac{\mathrm{p}_{\mathrm{i}}}{\mathrm{p}_{\mathrm{tot}}}=\frac{\mathrm{H}_{\mathrm{i}} \mathrm{x}_{\mathrm{i}}}{\mathrm{p}_{\mathrm{tot}}}$
Raoult's law. Used primarily for a component whose mole fraction approaches unity or for solutions of components quite similar in chemical nature, such as straight chain hydrocarbons.
$\mathrm{p}_{\mathrm{i}}=\mathrm{p}_{\mathrm{i}}^{*} \mathrm{x}_{\mathrm{i}}$
$\mathrm{p}_{\mathrm{i}}$ be the partial pressure of component i in the gas phase, $\mathrm{y}_{\mathrm{i}}$ be the gasphase mole fraction, and $x_{i}$ be the liquid-phase mole fraction.

Note that in the limit where $\mathrm{x}_{\mathrm{i}}=1, \mathrm{p}_{\mathrm{i}}=\mathrm{p}_{\mathrm{i}}{ }^{*}$

## Equilibrium constant $\mathbf{K}_{\mathbf{i}}$

$$
\mathrm{K}_{\mathrm{i}}=\frac{\mathrm{y}_{\mathrm{i}}}{\mathrm{x}_{\mathrm{i}}}=\frac{\mathrm{p}_{\mathrm{i}}^{*}}{\mathrm{p}_{\mathrm{tot}}}
$$

This Equation gives reasonable estimates of $\mathrm{K}_{\mathrm{i}}$ values at low pressures for components well below their critical temperatures.

## Example 12

A gas containing nitrogen, benzene, and toluene is in equilibrium with 40 mole\% benzene and 60 mole\% toluene liquid mixtures at $100^{\circ} \mathrm{C}$ and 10 atm. Estimate the gas phase composition (mole fractions) using Raoult's law.

## Solution

$$
\begin{aligned}
& \ln p^{*}=A-\frac{B}{C+T} \\
& \mathrm{p}_{\mathrm{B}}^{*}=\operatorname{Exp}\left[15.9008-\frac{2788.51}{-52.36+373}\right]=1344.9 \mathrm{mmHg}
\end{aligned}
$$

$$
\mathrm{p}_{\mathrm{T}}^{*}=\operatorname{Exp}\left[16.0137-\frac{3096.52}{-53.67+373}\right]=553.8 \mathrm{mmHg}
$$

Raoult's law, $\mathrm{p}_{\mathrm{i}}=\mathrm{p}_{\mathrm{i}}^{*} \mathrm{x}_{\mathrm{i}}$ and $\mathrm{p}_{\mathrm{i}}=\mathrm{y}_{\mathrm{i}} p_{\text {tot }}$
$y_{B} p_{t}=p_{B}^{*} X_{B}$
$y_{B}=\frac{0.4 * 1344.9}{(10)(760)}=0.071$
$\mathrm{y}_{\mathrm{T}}=\frac{0.6 * 553.8}{(10)(760)}=0.044$
$\mathrm{y}_{\mathrm{N} 2}=1-0.071-0.044=0.885$

## Example 13

Air and liquid water are contained at equilibrium in a closed chamber at $75^{\circ} \mathrm{C}$ and 760 mm Hg . Calculate the molar composition of the gas phase. p* ${ }^{\text {ноо }}\left(75^{\circ} \mathrm{C}\right)=289 \mathrm{~mm} \mathrm{Hg}$.

## Solution

Since the gas and liquid are in equilibrium, the air must be saturated with water vapor (if it was not, more water would evaporate), so that Raoult's law may be applied:
$y_{\mathrm{H} 2 \mathrm{O}}=\frac{\mathrm{p}_{\mathrm{H} 2 \mathrm{O}}^{*}}{\mathrm{p}}=\frac{289 \mathrm{mmHg}}{760 \mathrm{mmHg}}=0.38$
$y_{\text {dry air }}=1-$ ун2O $=1-0.38=0.62$

## Problems

1. An oxygen cylinder contains $1.0 \mathrm{ft}^{3}$ of $\mathrm{O}_{2}$ at $70^{\circ} \mathrm{F}$ and 200 psig . Calculate the volume of this $\mathrm{O}_{2}$ in a dry-gas holder at $90^{\circ} \mathrm{F}$ and 4.0 in. $\mathrm{H}_{2} \mathrm{O}$ above atmospheric. The barometer reads 29.92 in. Hg .
2. A cylinder has a volume of $1.0 \mathrm{ft}^{3}$ and contains dry methane at $80^{\circ} \mathrm{F}$ and 200 psig . What weight of methane is in the cylinder? The barometer pressure is 29.0 in . Hg .
3. What weight of ethane is contained in a gas cylinder that is 1.0 $\mathrm{ft}^{3}$ in volume if the gas is at $100{ }^{\circ} \mathrm{F}$ and 2000 psig? Solve this problem by Van der Waals' equation.
4. A large chamber contains dry $\mathrm{N}_{2}$ at $27^{\circ} \mathrm{C}$ and 101.3 kPa . Water is injected into the chamber. After saturation of the $\mathrm{N}_{2}$ with water vapor, the temperature in the chamber is $27^{\circ} \mathrm{C}$.
a-What is the pressure inside the chamber after saturation?
b-How many moles of $\mathrm{H}_{2} \mathrm{O}$ per moles of $\mathrm{N}_{2}$ are present in the saturated mixture?
5. Water in an enclosed vessel at $17^{\circ} \mathrm{C}$ contains a concentration of dissolved oxygen of $6 \mathrm{mg} / \mathrm{L}$. At equilibrium, determine the concentration of oxygen in the air space above the water in $\mathrm{mg} / \mathrm{L}$. Henry's law constant is $4.02 * 10^{6} \mathrm{kPa} / \mathrm{mol}$ fraction and the pressure in the vessel is 1 atm .

## Lecture 18

1. Energy Balances: Terminology And Thermodynamic Law

- A system is an object or a collection of objects that an analysis is carried out on. The system has a definite boundary, called the system boundary.
- Once a system is defined, through the choice of a system boundary, everything external to it is called the surroundings.
- All energy and material that are transferred out of the system enter the surroundings, and vice versa.
- An isolated system is a system that does not exchange heat, work, or material with the surroundings.
- A closed system is a system in which heat and work are exchanged across its boundary, but material is not.
- An open system can exchange heat, work, and material with the surroundings.


### 1.1.1 Forms of Energy: The First Law of Thermodynamics

- Energy is often categorized as kinetic energy, potential energy, and internal energy.
- Although energy cannot be created or destroyed, it can be converted from one form to another.
- Energy can also be transferred from one point to another or from one body to another one.
- Energy transfer can occur by flow of heat, by transport of mass, or by performance of work .
- The general energy balance for a thermodynamic process :

$$
\text { accumulation }=\text { input }- \text { output }
$$

- Energy can cross the boundaries of a closed system in the form of heat and work (Figure1.1).

The energy balance for a closed system takes the form:
$Q-W=\Delta U+\Delta K E+\Delta P E$
where heat (Q), work (W), internal energy (U), kinetic energy (KE), and potential energy (PE) are defined as follows.


FIGURE1.1 Energy balance for a closed system.

Work is the energy that flows in response to any driving force (e.g., applied force, torque) other than temperature, and is positive when the surroundings perform work on the system. Work is negative when the system performs work on the surroundings. In chemical processes, work may, for instance, come from pumps, compressors, moving pistons, and moving turbines.

Heat is the energy that flows due to a temperature difference between the system and its surroundings and always flows from regions at high temperatures to regions at low temperatures. By convention, heat is defined to be positive if it flows to a system (i.e., gained). For systems with no significant heat exchange with the surroundings, $\mathrm{Q}=0$. Such a system is said to be adiabatic.

An Isothermal system is one where the temperature does not change with time and in space.

## Kinetic Energy

Kinetic energy is the energy carried by a moving system because of its velocity.

- The kinetic energy KE of a moving object of mass m, traveling with speed $v$, is given by:
$K E=\frac{1}{2} m v^{2}$
Where KE in J.
- If a fluid enters a system with a mass flow rate ( $\mathrm{kg} / \mathrm{s}$ ) and uniform velocity ( $\mathrm{m} / \mathrm{s}$ ), then;
$\dot{K} \mathrm{E}=\frac{1}{2} \dot{\mathrm{~m}} \mathrm{v}^{2}$
Where KE in J/s.


## Example 1

Water flows into a process unit through a 2 cm ID pipe at a rate of $2 \mathrm{~m}^{3} / \mathrm{h}$. calculate the kinetic energy for this stream in joules/second.

## Solution

$$
\begin{aligned}
& \left.\mathrm{V}=\frac{2 \mathrm{~m}^{3}}{\mathrm{~h}}\left|\frac{100^{2} \mathrm{~cm}^{2}}{1^{2} \mathrm{~m}^{2}}\right| \frac{1}{\pi(1)^{2} \mathrm{~cm}^{2}} \right\rvert\, \frac{1 \mathrm{~h}}{3600 \mathrm{~s}}=1.77 \mathrm{~m} / \mathrm{s} \\
& \dot{\mathrm{~m}}=\frac{2 \mathrm{~m}^{3}}{\mathrm{~h}}\left|\frac{1000 \mathrm{~kg}}{\mathrm{~m}^{3}}\right| \frac{1 \mathrm{~h}}{3600 \mathrm{~s}}=0.556 \mathrm{~kg} / \mathrm{s} \\
& \dot{\mathrm{KE}}=\frac{0.556 \mathrm{~kg} / \mathrm{s}}{2}\left|\frac{(1.77)^{2} \mathrm{~m}^{2}}{\mathrm{~s}^{2}}\right| \frac{1 \mathrm{~N}}{1 \mathrm{~kg} \cdot \frac{\mathrm{~m}}{\mathrm{~s}^{2}}}=0.87 \mathrm{~N} \cdot \frac{\mathrm{~m}}{\mathrm{~s}}=0.87 \mathrm{~J} / \mathrm{s}
\end{aligned}
$$

## Potential Energy

Potential energy is the energy due to the position of the system in a potential field (e.g., earth's gravitational field, $g=9.81 \mathrm{~m} / \mathrm{s}^{2}$ ).

- For mass (m) at an elevation (h) in a gravitational field, relative to its gravitational potential energy at a reference elevation:

$$
\mathrm{PE}=\mathrm{mgh}
$$

Where PE in J.

- If a fluid enters a system with a mass flow rate and an elevation relative to the potential energy reference plane, then:
$\dot{\mathrm{PE}}=\dot{\mathrm{m}} \mathrm{gh}$
Where PE in J/s.
- Since we are normally interested in the change in potential energy when a body or fluid moves from one elevation to another:

$$
\dot{\mathrm{PE}}_{2}-\dot{\mathrm{PE}}_{1}=\dot{\mathrm{m} g}\left(\mathrm{~h}_{2}-h_{1}\right)
$$

## Example 2

Water is pumped from one reservoir to another 300 ft away. The water level in the second reservoir is 40 ft above the water level of the first reservoir. What is the increase in specific potential energy of the water in $\mathrm{Btu} / \mathrm{lb}_{\mathrm{m}}$ ?


## Solution

$\left.\Delta \widehat{\mathrm{PE}}=\frac{32 \mathrm{ft}}{\mathrm{s}^{2}}|(40-0) \mathrm{ft}| \frac{1\left(\mathrm{lb}_{\mathrm{f}}\right)\left(\mathrm{s}^{2}\right)}{32.2\left(\mathrm{lb}_{\mathrm{m}}\right)(\mathrm{ft})} \right\rvert\, \frac{1 \mathrm{Btu}}{778.2(\mathrm{ft})\left(\mathrm{lb}_{\mathrm{f}}\right)}=0.0514 \mathrm{Btu} / \mathrm{lb}_{\mathrm{m}}$

## Internal Energy

Internal energy (U) is a macroscopic concept that take into account all of the molecular, atomic, and subatomic energies.
$\Delta \widehat{U}=\widehat{U_{2}}-\widehat{U_{1}}=\int_{\widehat{U_{1}}}^{T_{2}} d \widehat{U}=\int_{T_{1}}^{T_{2}} C_{v} d T$

## Example 3

What is the change in internal energy when 10 kmol of air is cooled from $60{ }^{\circ} \mathrm{C}$ to $30^{\circ} \mathrm{C}$ in a constant volume process? Given that $\mathrm{C}_{\mathrm{v}}$ is $2.1 * 10^{4}$ $\mathrm{J} /(\mathrm{kmol})^{\circ} \mathrm{C}$.

## Solution

$\Delta \mathrm{U}=10 \mathrm{kmol} \int_{60^{\circ} \mathrm{C}}^{30^{\circ} \mathrm{C}}\left(2.1 * 10^{4} \frac{\mathrm{~J}}{(\mathrm{kmol})\left({ }^{\circ} \mathrm{C}\right)}\right) \mathrm{dT}=2.1 * 10^{5}(30-60)=$ $-6.3 * 10^{6} \mathrm{~J}$

## Enthalpy

From energy balance, we will find:
$H=U+p V$

The combined variables are called enthalpy. Where P is the pressure and V is the volume.
$\Delta \widehat{H}=\widehat{H_{2}}-\widehat{H_{1}}=\int_{\widehat{H_{1}}}^{\widehat{H_{2}}} d \widehat{H}=\int_{T_{1}}^{T_{2}} C_{p} d T$

As with internal energy, enthalpy has no absolute value, only changes in enthalpy can be calculated.

## Example 4

Calculate the enthalpy change when 10 kmol of air is cooled from 60 ${ }^{\circ} \mathrm{C}$ to $30{ }^{\circ} \mathrm{C}$ in a constant pressure process? Given that $\mathrm{C}_{\mathrm{p}}$ is $2.9 * 10^{4}$ $\mathrm{J} /(\mathrm{kmol})^{\circ} \mathrm{C}$.

## Solution

$\Delta \mathrm{H}=10 \mathrm{kmol} \int_{60^{\circ} \mathrm{C}}^{30^{\circ} \mathrm{C}}\left(2.9 * 10^{4} \frac{\mathrm{~J}}{(\mathrm{kmol})\left({ }^{\circ} \mathrm{C}\right)}\right) \mathrm{dT}=2.9 * 10^{5}(30-$
60) $=-8.7 * 10^{6} \mathrm{~J}$

## Lecture 19

## Energy balance for processes without chemical reaction

The principle of the conservation of energy ( or energy balance) states that the total energy of the system plus the surroundings can neither be created nor destroyed.

## 1. Energy balance for closed, unsteady state system

A batch process system is, by definition, closed, and semibatch and continuous systems are open. An integral energy balance may be derived for a closed system between two instants of time.
accumulation $=$ input - output
final system energy - initial system energy $=$ net energy transfered to the system
initial system energy $=U_{i}+K E_{i}+P E_{i}$
final system energy $=U_{f}+K_{f}+\mathrm{PE}_{\mathrm{f}}$
energy transfered $=\mathrm{Q}+\mathrm{W}$
$\left(U_{f}-U_{i}\right)+\left(K E E_{f}-K E_{i}\right)+\left(P E_{f}-P E_{i}\right)=Q+W$
$\Delta(\mathrm{U}+\mathrm{KE}+\mathrm{PE})=\mathrm{Q}+\mathrm{W}$
remember that Q and W are both positive when transferred into the system.
In general for closed system, $\triangle \mathrm{KE}$ and $\triangle \mathrm{PE}$ are zero, so the energy balance becomes:
$\Delta \mathrm{U}=\mathrm{Q}+\mathrm{W}$

## Example 5

A cylinder fitted with a movable piston is filled with gas. An amount of 2.00 kcal of heat is transferred to the gas to raise the gas temperature $100^{\circ} \mathrm{C}$ higher. The gas does 68 J of work in moving the piston to its new equilibrium position. Calculate the change in internal energy of the system.


## Solution

Because No change in kinetic and potential energy; accordingly, both are set to zero. The equation is reduced to
$\Delta \mathrm{U}=\mathrm{Q}-\mathrm{W}$
$\Delta \mathrm{U}=(2 \mathrm{kcal})\left[\frac{1000 \mathrm{cal}}{\text { kcal }} \left\lvert\, \frac{1 \mathrm{~J}}{0.239 \mathrm{cal}}\right.\right]-68 \mathrm{~J}=8300 \mathrm{~J}$

## 2. Energy balance for closed, steady state system

$\Delta \mathrm{KE}=0, \Delta \mathrm{U}=0$
$\Delta \mathrm{PE}=0, \Delta \mathrm{E}=0$

Then:

$$
\begin{aligned}
& \mathrm{Q}+\mathrm{W}=0 \\
& \mathrm{~W}=-\mathrm{Q}
\end{aligned}
$$

Meaning that all of the work done on a closed, steady-state system must be transferred out as heat (-Q). However, the reverse is false.

## 3. Energy balance for open, unsteady state systems

$\Delta \mathrm{E}=\left(\widehat{\mathrm{U}}_{1}+\widehat{\mathrm{KE}}_{1}+\widehat{\mathrm{PE}}_{1}\right) \mathrm{m}_{1}-\left(\widehat{\mathrm{U}}_{2}+\widehat{\mathrm{KE}}_{2}+\widehat{\mathrm{PE}}_{2}\right) \mathrm{m}_{2}+\mathrm{Q}+\mathrm{W}+$ $\mathrm{p}_{1} \widehat{\mathrm{~V}}_{1} \mathrm{~m}_{1}-\mathrm{p}_{2} \widehat{\mathrm{~V}}_{2} \mathrm{~m}_{2}$

Or,
$\Delta \mathrm{E}=\left(\widehat{\mathrm{H}}_{1}+\widehat{\mathrm{KE}}_{1}+\widehat{\mathrm{PE}}_{1}\right) \mathrm{m}_{1}-\left(\widehat{\mathrm{H}}_{2}+\widehat{\mathrm{KE}}_{2}+\widehat{\mathrm{PE}}_{2}\right) \mathrm{m}_{2}+\mathrm{Q}+\mathrm{W}$

## Example 6:

The specific internal energy of helium at 300 K and 1 atm is 3800 $\mathrm{J} / \mathrm{mol}$, and the specific molar volume at the same temperature and pressure is $24.63 \mathrm{~L} / \mathrm{mol}$. Calculate the specific enthalpy of helium at this temperature and pressure.

## Solution

$$
\widehat{\mathrm{H}}=\widehat{\mathrm{U}}+\mathrm{P} \widehat{\mathrm{~V}}=3800 \frac{\mathrm{~J}}{\mathrm{~mol}}+(1 \mathrm{~atm})\left(24.63 \frac{\mathrm{~L}}{\mathrm{~mol}}\right)
$$

Since $0.082 \mathrm{~L} . \mathrm{atm} / \mathrm{mol} \mathrm{K}=8.314 \mathrm{~J} / \mathrm{mol} \mathrm{K}$

$$
\begin{aligned}
& \frac{8.314 \frac{\mathrm{~J}}{\mathrm{~mol} \mathrm{~K}}}{0.082 \frac{\mathrm{Lamm}}{\text { mol } \mathrm{K}}}=101.3 \frac{\mathrm{~J}}{\mathrm{L.atm}} \\
& \left.\quad \widehat{\mathrm{H}}=3800 \frac{\mathrm{~J}}{\mathrm{~mol}}+\left(24.63 \frac{\mathrm{~L} \cdot \mathrm{~atm}}{\mathrm{~mol}}\right) \right\rvert\, \frac{101.3 \mathrm{~J}}{1 \mathrm{~L} . \mathrm{atm}}=6295 \mathrm{~J} / \mathrm{mol}
\end{aligned}
$$

## 4. Energy balance for open, steady state systems

$$
\mathrm{Q}+\mathrm{W}=\Delta(\mathrm{H}+\mathrm{KE}+\mathrm{PE})
$$

And because $\triangle \mathrm{KE}$ and $\triangle \mathrm{PE}$ are always equal zero, $\mathrm{Q}+\mathrm{W}=\Delta \mathrm{H}$

## Example 6

Five hundred kilograms per hour of steam drives a turbine. The steam enters the turbine at 44 atm and 450 C at a linear velocity of $60 \mathrm{~m} / \mathrm{s}$ and leaves at a point 5 m below the turbine inlet at atmospheric pressure and a velocity of $360 \mathrm{~m} / \mathrm{s}$. The turbine delivers shaft work at a rate of 70 kW , and the heat loss from the turbine is estimated to be $10 \mathrm{kcal} / \mathrm{h}$. Calculate the specific enthalpy change associated with the process.

## Solution


$\Delta \dot{\mathrm{H}}=\dot{\mathrm{Q}}-\dot{\mathrm{W}}_{\mathrm{S}}-\Delta \dot{\mathrm{KE}}-\Delta \dot{\mathrm{PE}}$
$\Delta \dot{\mathrm{K}} \mathrm{E}=\frac{1}{2} \dot{\mathrm{~m}}\left(\mathrm{v}_{2}{ }^{2}-\mathrm{v}_{1}{ }^{2}\right)=\frac{1}{2} 0.139 \mathrm{~kg} / \mathrm{s}\left|\frac{1 \mathrm{~N}}{1 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{s}^{2}}\right|\left(360^{2}-\right.$ $\left.60^{2}\right) \frac{\mathrm{m}^{2}}{\mathrm{~s}^{2}}\left|\frac{1 \mathrm{~W}}{1 \mathrm{~N} \cdot \frac{\mathrm{~m}}{\mathrm{~s}}}\right| \frac{1 \mathrm{~kW}}{10^{3} \mathrm{~W}}=8.75 \mathrm{~kW}$
$\left.\Delta \dot{\mathrm{PE}}=\dot{\operatorname{m} g}\left(\mathrm{~h}_{2}-h_{1}\right)=\frac{0.139 \mathrm{~kg}}{s}\left|\frac{9.81 \mathrm{~N}}{\mathrm{~kg}}\right|(-5) m \right\rvert\, \frac{1 \mathrm{~kW}}{10^{3} \mathrm{~N} \cdot \frac{\mathrm{~m}}{s}}=$ $-6.81 * 10^{-3} \mathrm{~kW}$
$\left.\dot{\mathrm{Q}}=\frac{-10^{4} \mathrm{kcal}}{\mathrm{h}}\left|\frac{1 \mathrm{~J}}{0.239 * 10^{-3} \mathrm{kcal}}\right| \frac{1 \mathrm{~h}}{3600 \mathrm{~s}} \right\rvert\, \frac{1 \mathrm{~kW}}{\frac{\mathrm{k}^{3} \mathrm{~J}}{\mathrm{~s}}}=-11.6 \mathrm{~kW}$
$\Delta \dot{\mathrm{H}}=-11.6-70-8.75-\left(-6.81 * 10^{-3}\right)=-90.3 \mathrm{~kW}$
$\Delta \widehat{\mathrm{H}}=\frac{\Delta \dot{\mathrm{H}}}{\dot{\mathrm{m}}}=\frac{-90.3 \mathrm{~kJ} / \mathrm{s}}{0.139 \mathrm{~kg} / \mathrm{s}}=-650 \mathrm{~kJ} / \mathrm{kg}$

## Example 7

Water is pumped from a well in which the water level is a constant 20 feet below the ground level. The water is discharged into a level pipe that is 5 feet above the ground at a rate of $0.5 \mathrm{ft}^{3} / \mathrm{s}$. Calculate the electric power required by the pump if it is $100 \%$ efficient.

## Solution

$\mathrm{Q}=0$ ( given assumption)
$\Delta \mathrm{KE}=0$ (negligible change in KE )
$\Delta \mathrm{H}=0$ because the temperature of water is the same in well and in discharge water.
The energy balance is reduced to :
$\mathrm{W}=\Delta \mathrm{PE}=\dot{\mathrm{m}} \mathrm{g}\left(\mathrm{h}_{\text {out }}-\mathrm{h}_{\mathrm{in}}\right)$

Basis: 1 second

$$
\dot{m}=\frac{0.5 \mathrm{ft}^{3}}{s} \left\lvert\, \frac{62.4 \mathrm{lb}_{m}}{f t^{3}}=31.3 \mathrm{lb}_{\mathrm{m}} / \mathrm{s}\right.
$$

$\mathrm{W}=\Delta \mathrm{PE}=$
$\left.31.3 \frac{\mathrm{lb}_{\mathrm{m}}}{\mathrm{s}}\left|\frac{32.2 \mathrm{ft}}{\mathrm{s}^{2}}(5-(-20) \mathrm{ft})\right| \frac{\mathrm{s}^{2} \mathrm{lb}_{\mathrm{f}}}{32.2 \mathrm{ft} \mathrm{lb}_{\mathrm{m}}} \right\rvert\, \frac{1.055 \mathrm{~kW} \mathrm{~s}}{778.2 \mathrm{lb}_{\mathrm{f}} \mathrm{ft}}=1.06 \mathrm{~kW}$

## Problems

1. Write and simplify the closed-system energy balance for each of the following processes:
a. A tray filled with water at $20^{\circ} \mathrm{C}$ is put into a freezer. The water turns into ice at $-5^{\circ} \mathrm{C}$.
b. A chemical reaction takes place in a closed adiabatic rigid container.
2. Oxygen at 150 K and 41.64 atm has a tabulated specific volume of $4.684 \mathrm{~cm}^{3} / \mathrm{g}$ and a specific internal energy of $1706 \mathrm{~J} / \mathrm{mol}$. Calculate the specific enthalpy of $\mathrm{O}_{2}$ in this state.
3. Prove that for an ideal gas, $\widehat{U}$ and $\widehat{H}$ are related as $\widehat{H}=\widehat{U}+R T$ where R is the gas constant. Then calculate $\Delta \mathrm{H}(\mathrm{cal})$ for a process in which the temperature of 2.5 mol of an ideal gas is raised by 50 ${ }^{\circ} \mathrm{C}$, resulting in a specific internal energy of $3500 \mathrm{cal} / \mathrm{mol}$.
4. Air is heated from $25^{\circ} \mathrm{C}$ to $150^{\circ} \mathrm{C}$ prior to entering a combustion furnace. The change in specific enthalpy associated with this transition is $3640 \mathrm{~J} / \mathrm{mol}$. The flow rate of air at the heater outlet is $1.25 \mathrm{~m}^{3} / \mathrm{min}$ and the air pressure at this point is 122 kPa absolute. Calculate the heat requirement in kW .(make your assumptions)
5. Steam at $260^{\circ} \mathrm{C}$ and 7 bar absolute is expanded through a nozzle to $200^{\circ} \mathrm{C}$ and 4 bar. Negligible heat is transferred from the nozzle to its surroundings. The approach velocity of the steam is negligible. The specific enthalpy of steam is $2974 \mathrm{~kJ} / \mathrm{kg}$ at $260^{\circ} \mathrm{C}$ and 7 bar and $2860 \mathrm{~kJ} / \mathrm{kg}$ at $200^{\circ} \mathrm{C}$ and 4 bar. Calculate the exit steam velocity.
6. Air is being compressed from 100 kPa and 255 K (where it has an enthalpy of $489 \mathrm{~kJ} / \mathrm{kg}$ ) to 1000 kPa and 278 K (where it has an enthalpy of $509 \mathrm{~kJ} / \mathrm{kg}$ ). the exit velocity of the air from compressor is $60 \mathrm{~m} / \mathrm{s}$. What is the power required (in kW ) for the compressor if the load is $100 \mathrm{~kg} / \mathrm{h}$ of air?

## Lecture 20

## Energy balance for processes with chemical reaction

For continuous, steady state processes, the general energy balance reduces to two choices:

- With the effects of chemical reactions merged with the sensible heats:

$$
\begin{aligned}
& Q=\Delta H=\left[H(T)-H\left(25^{\circ} C\right)\right]_{\text {outputs }}-\left[H(T)-H\left(25^{\circ} C\right)\right]_{\text {inputs }} \\
& =\Delta H_{\text {output }}-\Delta H_{\text {input }}---------------------(1)
\end{aligned}
$$

- With the effects of chemical reaction lumped in the heat of reaction

$$
\begin{align*}
& Q=\left[H(T)-H\left(25^{\circ} C\right)\right]^{\text {sensible }+ \text { phase change }} \text { outputs }- \\
& {\left[H(T)-H\left(25^{\circ} C\right)\right]^{\text {sensible }+ \text { phase change }_{\text {inputs }}+\Delta H_{r x n}}} \tag{2}
\end{align*}
$$

Here are some application of the above equation:
What is the temperature of one stream given data for other streams?
\$ How much heat has to be added to or removed from the process?
What is the temperature of the reaction?
How much material must be added or removed from the process to give a specified value of heat transfer. We focus how to determine the heat added or removed due to chemical reaction.

For the following reaction :
$a A+b B \rightarrow c C+d D$
The heat of reaction, $\Delta \mathrm{H}_{\mathrm{Rx}}(\mathrm{T}, \mathrm{P})$, is the enthalpy change for a process in which stoichiometric quantities of reactants at temperature T and pressure P react completely to form products at the same temperature and pressure.

The standard heat of reaction $\left(\Delta \mathrm{H}_{\mathrm{Rx}}{ }^{\circ}\right)$ is calculated as the difference between the product and reactant enthalpies when both reactants and products are at standard conditions, that is, at $25^{\circ} \mathrm{C}$ and 1 atm. The symbol " 0 " denotes standard conditions. Therefore,
$\Delta H_{R x}^{\circ}\left[\frac{\mathrm{kJ}}{\mathrm{mol}}\right]=H_{\text {products }}-H_{\text {reactants }}$
$=c \Delta H_{f, C}^{o}+d \Delta H_{f, D}^{o}-a \Delta H_{f, A}^{o}-b \Delta H_{f, B}^{o}$
$=\sum v_{i} \Delta H_{f, i}^{o}$
where $\Delta \mathrm{H}_{\mathrm{f}}{ }^{0}$ is the standard heat of formation.
Also ,
$\Delta H_{R x}^{o}=-\sum v_{i}\left(\Delta H_{c}^{o}\right)_{i}$
Where $\left(\Delta \mathrm{H}_{\mathrm{C}}{ }^{0}\right)_{\mathrm{i}}$ is the standard heat of combustion of species i. If any reactants or products are combustion products (i.e., $\mathrm{CO}_{2}, \mathrm{H}_{2} \mathrm{O}, \mathrm{SO}_{2}$ ), their heats of combustion are equal to zero.

The standard enthalpy of combustion is the enthalpy change when 1 mol of a reactant completely burns in excess oxygen under standard thermodynamic conditions.

## Example 8

Consider the following reaction:

$$
\mathrm{A}+2 \mathrm{~B} \rightarrow 3 \mathrm{C}, \quad \mathrm{HR}_{\mathrm{x}}\left(100^{\circ} \mathrm{C}, 1 \mathrm{~atm}\right)=-50 \mathrm{~kJ} / \mathrm{mol}
$$

The enthalpy change for the given reaction is;
$\frac{-50 \mathrm{~kJ}}{1 \text { mol A consumed }}=\frac{-50 \mathrm{~kJ}}{2 \text { mol } B \text { consumed }}=\frac{-50 \mathrm{~kJ}}{3 \text { mol C generated }}$
If $150 \mathrm{~mol} / \mathrm{s}$ of C was generated at $100^{\circ} \mathrm{C}$ and 1 atm , then:
$\Delta \dot{H}=\left(\frac{-50 \mathrm{~kJ}}{3 \text { mol C generated }}\right)\left(\frac{150 \mathrm{~mol} \text { C generated }}{\mathrm{s}}\right)=2500 \mathrm{~kJ} / \mathrm{s}$
Or :
$\Delta \dot{H}=\dot{\xi} \Delta H_{R x}(T, P)$
If $\Delta H_{R x}(T)$ is negative, the reaction is exothermic; that is, energy must be removed from the reactor to prevent the temperature from increasing. If
$\Delta H_{R x}(T)$ is positive, the reaction is endothermic; that is, energy must be added to the reactor to prevent the temperature from decreasing.

## Example 9

Consider the combustion of liquid ethanol as shown in the following reaction scheme:
$\mathrm{C}_{2} \mathrm{H}_{5} \mathrm{OH}(\mathrm{l})+3 \mathrm{O}_{2}(\mathrm{~g}) \rightarrow 2 \mathrm{CO}_{2}(\mathrm{~g})+3 \mathrm{H}_{2} \mathrm{O}(\mathrm{l})$
Use heat of formation and heat of combustion to determine the standard heat of reaction.

## Solution

Values for standard heat of combustion and standard heat of formation are available in the Tables.

$$
\begin{aligned}
& \Delta H_{R x}^{\circ}=3 \Delta H_{f, H 2 O(l)}^{\circ}+2 \Delta H_{f, C O 2}^{\circ}-0-\Delta H_{f, C 2 H 5 O H_{(l)}^{\circ}} \\
& \Delta H_{R x}^{\circ}=3(-285.84)+2(-393.51)-0-(-277.63)=-1366.9 \mathrm{~kJ} / \\
& \mathrm{mol}
\end{aligned}
$$

The standard heat of reaction is also calculated from the standard heat of combustion as:

$$
\Delta H_{R x}^{\circ}=\Delta H_{c, C 2 H 5 O H(l)}^{\circ}+3 \Delta H_{c, 02}^{\circ}-3 \Delta H_{c, H 2 O(l)}^{\circ}-2 \Delta H_{c, C O 2_{(g)}^{\circ}}
$$

The standard heat of combustion of oxygen, water, and carbon dioxide are zero:
$\Delta H_{R x}^{\circ}=-1366.91+0-0-0=-1366.9 \mathrm{~kJ} / \mathrm{mol}$
Results reveal that both values of standard heat of reactions are identical.

* For a single reaction at a reference state of $25^{\circ} \mathrm{C}$ and 1 atm while reactant and product are at different inlet and exit temperatures:

$$
\Delta \dot{H}=\dot{\xi} \Delta H_{R x}^{\circ}+\sum_{\text {out }} \dot{n}_{i} \bar{h}_{i}-\sum_{\text {in }} \dot{n}_{i} \bar{h}_{i}
$$

where $h_{i}(\mathrm{~J} / \mathrm{mol})$ is the specific molar enthalpy of a definite component.

## Example 10

Consider the following gas phase reaction :
$\mathrm{CO}_{2}(\mathrm{~g})+4 \mathrm{H}_{2}(\mathrm{~g}) \rightarrow 2 \mathrm{H}_{2} \mathrm{O}(\mathrm{g})+\mathrm{CH}_{4}(\mathrm{~g})$
Which proceed with $100 \%$ conversion of $\mathrm{CO}_{2}$. Assume stoichiometric amounts of the reactants enter the reactor. Determine the heat of reaction if the gases enter and leave at 1 atm and $500^{\circ} \mathrm{C}$. given that:

| Compound | $\boldsymbol{\Delta} \widehat{\boldsymbol{H}}_{\boldsymbol{f}}^{\boldsymbol{o}}\left(\frac{\boldsymbol{k J}}{\mathbf{m o l}}\right)$ | $\boldsymbol{\Delta} \widehat{\boldsymbol{H}}_{\mathbf{2 5 0} \mathbf{0} \mathbf{C}}{ }^{\circ} \mathrm{C}\left(\frac{\boldsymbol{k J}}{\mathbf{m o l}}\right)$ |
| :--- | :--- | :--- |
| $\mathrm{CO}_{2}(\mathrm{~g})$ | -393.25 | 21.425 |
| $\mathrm{H}_{2}(\mathrm{~g})$ | 0 | 13.834 |
| $\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ | -241.835 | 17.010 |
| $\mathrm{CH}_{4}(\mathrm{~g})$ | -74.848 | 23.126 |



## Solution

Basis: $1 \mathrm{gmol} \mathrm{CO}_{2}(\mathrm{~g})$ at 1 atm and $500^{\circ} \mathrm{C}$
$\Delta H=\xi \Delta H_{R x}^{\circ}+\sum_{\text {out }} n_{i} \bar{h}_{i}-\sum_{\text {in }} n_{i} \bar{h}_{i}$
$\Delta H_{R x}^{\circ}\left[\frac{\mathrm{kJ}}{\mathrm{mol}}\right]=H_{\text {products }}-H_{\text {reactant }}=\sum v_{i} \Delta H_{f, i}^{\circ}$

$$
\begin{aligned}
\Delta H_{R x}^{\circ} & =[(1)(-74.848)+2(-241.835)]-[(1)(-393.25)+4(0)] \\
& =-165.27 \mathrm{~kJ}
\end{aligned}
$$

$$
\sum_{i}^{\text {input }}\left[H\left(500^{\circ} \mathrm{C}\right)-H\left(25^{\circ} \mathrm{C}\right)\right]=[(1)(21.425)+4(13.834)]
$$

$$
=76.761 \mathrm{~kJ}
$$

$$
\sum_{i}^{\text {output }}\left[H\left(500^{\circ} \mathrm{C}\right)-H\left(25^{\circ} \mathrm{C}\right)\right]=[(2)(17.010)+1(23.126)]
$$

$$
=57.146 \mathrm{~kJ}
$$

$\Delta \mathrm{H}\left(500^{\circ} \mathrm{C}\right)=(-165.27)+57.146-76.761=-184.9 \mathrm{~kJ}$

## Example 11

Consider the same reaction of example 3 , but $\mathrm{CO}_{2}$ entering from another process at 800 K , and reacting with $4 \mathrm{gmol} \mathrm{H}_{2}$ entering at 298 K , that only $70 \%$ conversion of $\mathrm{CO}_{2}$ occurred. The products exited from the reactor at 1000 K . Calculate the heat transfer to or from the reactor. Given that;

| Compound | Temperatute K |  | $\Delta \widehat{H}^{\mathbf{2 5}{ }^{\boldsymbol{o}} \mathrm{C}}\left(\frac{\mathrm{kJ}}{\mathrm{mol}}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{CO}_{2}(\mathrm{~g})$ | 800 | 1000 | 22.798 | 33.396 |
| $\mathrm{H}_{2}$ (g) | 298 | 1000 | 0 | 20.62 |
| $\mathrm{H}_{2} \mathrm{O}(\mathrm{g})$ | 1000 |  | 25.986 |  |
| $\mathrm{CH}_{4}$ (g) | 1000 |  | 38.325 |  |

## Solution

The system is steady state and open with reaction. Assume 1 atm.
Basis: 1 gmol of $\mathrm{CO}_{2}$ (g)
The reference temperature is $25^{\circ} \mathrm{C}$.
$\mathrm{CO}_{2}(g)+4 \mathrm{H}_{2}(g) \rightarrow 2 \mathrm{H}_{2} \mathrm{O}(g)+\mathrm{CH}_{4}(g)$
$\Delta H=\xi \Delta H_{R x}^{o}+\sum_{\text {out }} n_{i} \bar{h}_{i}-\sum_{\text {in }} n_{i} \bar{h}_{i}$
The first step is to calculate the heat of reaction at the reference temperature.

$$
\begin{aligned}
\Delta H_{R x}^{\circ} & =[(1)(-74.848)+2(-241.835)]-[(1)(-393.25)+4(0)] \\
& =-165.27 \mathrm{~kJ}
\end{aligned}
$$

For $70 \%$ conversion of the $\mathrm{CO}_{2}$ :
$\Delta \mathrm{H}_{\mathrm{Rx}}^{\circ}=(0.7)(-165.27)=-115.69 \mathrm{~kJ}$
The next step is to calculate the enthalpy changes from 298 K to the respective temperatures of the compounds entering and leaving the reactor.

$$
n_{C O 2}=1+(0.7)(-1)=0.3 \mathrm{~mol}
$$

$$
\begin{aligned}
& n_{H_{2}}=4+(0.7)(-4)=1.2 \mathrm{~mol} \\
& n_{H_{2} O}=0+(0.7)(2)=1.4 \mathrm{~mol} \\
& n_{C H_{4}}=0+(0.7)(1)=0.7 \mathrm{~mol} \\
& \sum_{\mathrm{i}}^{\text {input }}\left[\mathrm{H}(\mathrm{~T})-\mathrm{H}\left(25^{\circ} \mathrm{C}\right)\right]=[(1)(22.798)+4(0)]=22.798 \mathrm{~kJ} \\
& \sum_{\mathrm{i}}^{\text {output }}\left[\mathrm{H}(\mathrm{~T})-\mathrm{H}\left(25^{\circ} \mathrm{C}\right)\right]=[(1.4)(25.986)+(0.7)(38.325)+ \\
& (0.3)(33.396)+(1.2)(20.62)]=97.971 \mathrm{~kJ}
\end{aligned}
$$

The energy balance reduces to $\mathrm{Q}=\Delta \mathrm{H}$

$$
\mathrm{Q}=\Delta \mathrm{H}=(-115.69)+(97.971)-(22.798)=-40.517 \mathrm{~kJ}
$$

## Lecture 21

## Ideal Processes, Efficiency, And The Mechanical Energy Balance

The ideal processes is a hypothetical process that rarely occurs in practice. Why bother with it then? The reason is that the calculations for energy changes can be made for an ideal process, and then an empirical efficiency used to convert the ideal work or energy change into the actual work or energy change.

## 1. Ideal reversible processes

Reversible process is an idealized process (hypothetical) in which changes occur because of an infinitesimal (differential) imbalance( اختلال متناهي الصغر (تفاضلي)) of temperature, pressure, and so on.
2. Irreversible process

A process that is not reversible because it occurs with finite differences(اختلافات محدودة) of temperature, pressure, etc.- most real processes.

## Example 1

How much work is done by 1 liter of saturated liquid water when it evaporates from an open vessel into the atmosphere where the pressure is 100 kPa ?

## Solution



Basis : 1 liter of water (liquid)
Open system and reversible process.
The unknown quantity is the work done by the liquid water pushing against the atmosphere. The general energy balance:
$\Delta \mathrm{E}=\mathrm{Q}+\mathrm{W}-\Delta[(\widehat{\mathrm{H}}+\widehat{\mathrm{KE}}+\widehat{\mathrm{PE}}) \mathrm{m}]$
Will not be useful because Q is unknown. We can imagine that an expandable bag is placed over the open face of the vessel so that the system comprised of the water becomes a closed system.

Because reversible process, the work for a closed system is:
$\mathrm{W}=-\int_{\mathrm{V}_{1}}^{\mathrm{V}_{2}} \mathrm{pdV}=-\mathrm{p} \Delta \mathrm{V}$
The specific volume of the liquid water from the SI steam tables is $0.001043 \mathrm{~m}^{3} / \mathrm{kg}$. The final volume of the water vapor is:

1 L liquid $\left.\left|\frac{1 \mathrm{~m}^{3}}{1000 \mathrm{~L}}\right| \frac{1 \mathrm{~kg}}{0.001043 \mathrm{~m}^{3} \text { liquid }} \right\rvert\, \frac{1.694 \mathrm{~m}^{3} \text { vapor }}{\mathrm{kg}}=1.624 \mathrm{~m}^{3}$ vapor
$\left.\mathrm{W}=-100 * 10^{3} \mathrm{~Pa}\left|\frac{1 \mathrm{~N} / \mathrm{m}^{2}}{1 \mathrm{~Pa}}\right| \frac{1 \mathrm{~J}}{1 \mathrm{~N} . \mathrm{m}} \right\rvert\, 1.624 \mathrm{~m}^{3}=-1.624 * 10^{5} \mathrm{~J}$

## Efficiency

Efficiency, in general, is the ratio of the work done by machine to the amount of fuel consumed and to the energy expended.

## General efficiency:

$\eta_{1}=\frac{\text { useful energy out }}{\text { energy in }}$

## Mechanical efficiency:

$\eta_{2}=\frac{\text { actual work output from the process }}{\text { work output for a reversible process }}$

## Heat engine efficiency:

$\eta_{4}=\frac{\text { actual work output from the process }}{\text { heat input from a source }}$

## Example 2

A small hydroelectric plant puts out 20 MW when the level of water above the generators is 25 m and the water flow rate through the generators is $100 \mathrm{~m}^{3} / \mathrm{s}$. What is the overall efficiency of the plant?

## Solution

the mass flow rate of the water is:
$\frac{100 \mathrm{~m}^{3}}{\mathrm{~s}} \left\lvert\, \frac{1000 \mathrm{~kg}}{\mathrm{~m}^{3}}=10^{5} \mathrm{~kg} / \mathrm{s}\right.$
The potential energy change of the water per second is:
$\Delta \dot{P E}=\dot{m} g h=\frac{10^{5} \mathrm{~kg}}{\mathrm{~s}}\left|9.807 \frac{\mathrm{~m}}{\mathrm{~s}^{2}}\right| 25 \mathrm{~m}\left|\frac{N s^{2}}{1 \mathrm{~kg} \cdot \mathrm{~m}}\right| \frac{\mathrm{s} \cdot \mathrm{W}}{1 \mathrm{~J}}=2.45 * 10^{7} \mathrm{~W}$
$\eta_{1}=\frac{\text { useful energy out }}{\text { energy in }}$
$\eta_{1}=\frac{20 * 10^{6} \mathrm{~W}}{2.45 * 10^{7} \mathrm{~W}}=0.82$

## The mechanical energy balance

In some processes, such as the compression of gases and pumping of liquids, work and the mechanical forms of energy(kinetic and potential energies) are the important factors. For these processes, an energy balance, including solely the mechanical forms of energy, becomes a useful tool.

The steady state mechanical energy balance is:
$\Delta(\widehat{\mathrm{KE}}+\widehat{\mathrm{PE}})+\int_{\mathrm{p}_{1}}^{\mathrm{p}_{2}} \widehat{\mathrm{~V}} \mathrm{dp}-\widehat{\mathrm{W}}+\widehat{\mathrm{E}}_{\mathrm{V}}=0$
Where $E_{v}$ is the loss of mechanical energy into nonmechanical forms of energy.

For a reversible process in which $\mathrm{E}_{\mathrm{v}}=0, \mathrm{~W}=0$, and incompressible fluid, equation 1 becomes:
$\frac{\Delta p}{\rho}+\frac{\Delta v^{2}}{2}+g \Delta h=0$
Equation 2 is called Bernoulli equation.
If each terms is divided by $g$, the $u n t s$ of the equation are tho se of $\Delta h$, and are known as heads of fluid.

## Example 3

Calculate the work per minute required to pump 1 lb of water per minute from 100 psia and $80^{\circ} \mathrm{F}$ to 1000 psia and $100^{\circ} \mathrm{F}$. the exit stream is 10 ft above entrance stream.


## Solution

The system is a steady state process and the general mechanical energy balance is :

$$
\Delta(\widehat{\mathrm{KE}}+\widehat{\mathrm{PE}})+\int_{\mathrm{p}_{1}}^{\mathrm{p}_{2}} \widehat{\mathrm{~V}} \mathrm{dp}-\widehat{\mathrm{W}}+\widehat{\mathrm{E}}_{\mathrm{V}}=0
$$

We assume:

1. $\Delta \mathrm{KE}=0$
2. The process is reversible, so $\mathrm{E}_{\mathrm{v}}=0$
3. Pump efficiency is $100 \%$

So the general mechanical energy balance reduces to :
$\widehat{W}=\int_{\mathrm{p}_{1}}^{\mathrm{p}_{2}} \widehat{\mathrm{~V}} \mathrm{dp}+\Delta \widehat{\mathrm{PE}}$

Basis: 1 minute of operation
From steam table, the specific volume of liquid water can be taken to be $0.0161 \mathrm{ft}^{3} / \mathrm{lb}$.
$\left.\Delta \widehat{\mathrm{PE}}=10 \mathrm{ft}\left|32.2 \frac{\mathrm{ft}}{\mathrm{s}^{2}}\right| \frac{\mathrm{lb}_{\mathrm{f}} \cdot \mathrm{s}^{2}}{32.2 \mathrm{lb}_{\mathrm{m}} . \mathrm{ft}} \right\rvert\, \frac{1 \mathrm{Btu}}{778 \mathrm{ft} . \mathrm{lb}_{\mathrm{f}}}=0.0129 \mathrm{Btu} / \mathrm{lb}_{\mathrm{m}}$ $\int_{100}^{1000} 0.0161 \mathrm{dp}=$
$\left.0.0161 \frac{\mathrm{ft}^{3}}{\mathrm{lb}_{\mathrm{m}}}\left|\frac{(1000-100) l b_{f}}{i n .^{2}}\right|\left(\frac{12 \mathrm{in} .}{1 \mathrm{ft}}\right)^{2} \right\rvert\, \frac{1 \mathrm{Btu}}{778 \mathrm{ft} . l b_{f}}=2.68 \frac{\mathrm{Btu}}{l b_{m}}$
$\widehat{W}=2.68+0.0129=2.69 \frac{B t u}{l b_{m}}$

## Problems

1. Calculate the heat of reaction at SC for 1 g mol of $\mathrm{H}_{2}(\mathrm{~g})$ using the heat combustion data, and then calculate $\Delta \mathrm{H}_{\mathrm{rxn}}$ for $\mathrm{H}_{2}(\mathrm{~g})$ at $0^{\circ} \mathrm{C}$.

$\Delta \mathrm{H}^{\circ}{ }_{\mathrm{c}}(\mathrm{kJ} / \mathrm{g} \mathrm{mol}) \quad-285.84 \quad 0 \quad 0$ $\begin{array}{llll}\mathrm{C}_{\mathrm{p}}\left(\mathrm{J} / \mathrm{g}{ }^{\circ} \mathrm{C}\right) & 14.36 & 0.908 & 4.184\end{array}$
2. Formaldehyde can be made by the oxidation of methanol:

$$
\begin{array}{lllll}
\mathrm{CH}_{3} \mathrm{OH}(\mathrm{~g})+ & \frac{1}{2} \mathrm{O}_{2}(\mathrm{~g}) \longrightarrow & \mathrm{H}_{2} \mathrm{CO}(\mathrm{~g})+\mathrm{H}_{2} \mathrm{O}(\mathrm{~g}) \\
\Delta \hat{H}_{\mathrm{f}}^{0}(\mathrm{~kJ} / \mathrm{gmol}): & -201.25 & 0 & -115.89 & -241.826
\end{array}
$$

If stochiometric amounts of methanol and oxygen enter the reactor at $100{ }^{\circ} \mathrm{C}$, the reaction is complete, and the products leave the reactor at $200^{\circ} \mathrm{C}$, calculate the heat that is added or removed from the reactor per mole of methanol fed to the reactor.
3. Calculate the work done when 1 lb mol of water in an open vessel evaporates completely at $212{ }^{\circ} \mathrm{F}$. Express your result in Btu.
4. A motor is rated as 30 hp . However, the output horsepower of the motor is only 24.6 hp . What is the efficiency of the motor?
5. A power plant is as shown in figure. If the pump moves 100 $\mathrm{gal} / \mathrm{min}$ into the boiler with an overall efficiency of $40 \%$. Find the horsepower required for the pump. List all additional assumption required.

6. Water at $20^{\circ} \mathrm{C}$ is being pumped from a constant-head tank open to the atmosphere to an elevated tank kept at a constant pressure of 1150 kPa as shown in figure. If water is following in the 5 cm line at rate of $0.4 \mathrm{~m}^{3} / \mathrm{min}$, find:
(a) The rating of the pump in joules per kilogram being pumped
(b) The rating of the pump in joules per minute

The pump and motor have an overall efficiency of $70 \%$ and the energy loss in the line can be determined to be $60 \mathrm{~J} / \mathrm{kg}$ flowing.


